Abstraction-based Reduction of Input Size for Neural Networks[1]

Peter Backeman, **Edin Jelačić**, Cristina Seceleanu, Ning Xiong, and Tiberiu Seceleanu

Mälardalen University, School of Innovation, Design and Engineering, Västerås, Sweden

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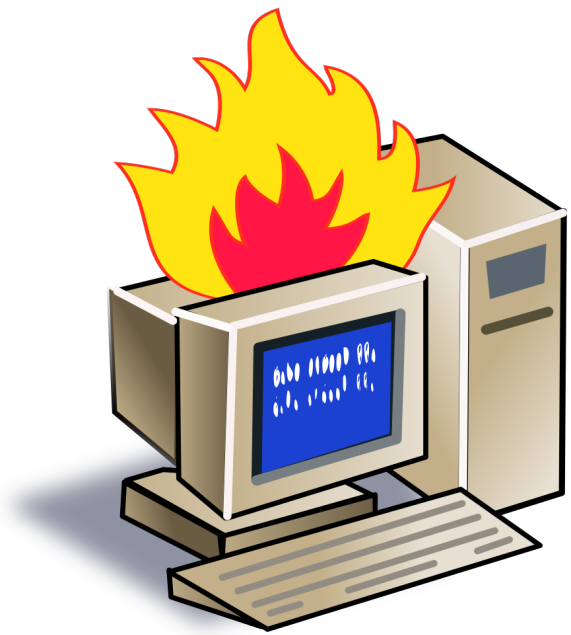
[1]Supported by the Knowledge Foundation

Use Case

Use Case: Predicting CPU overload.

- We have a set of functions $F = \{f_1, \ldots, f_n\}$.
- For a particular subset of F (a configuration), we can measure a CPU load.
- Question: for what configurations is there risk for an overload?



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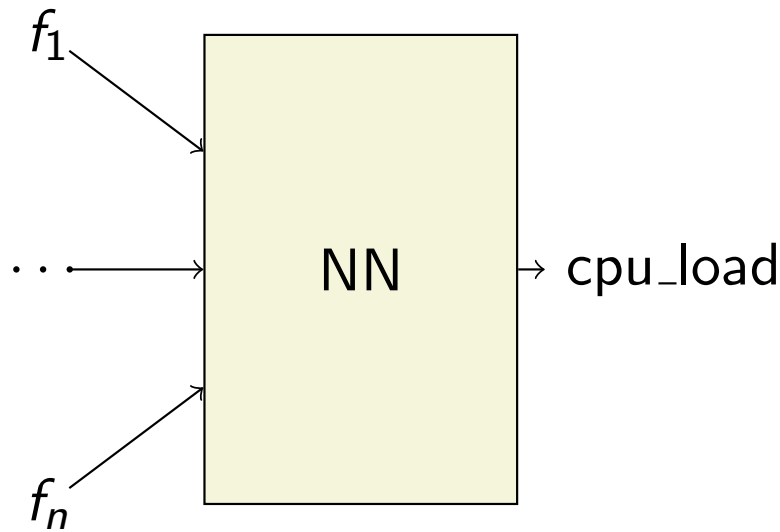
Modeling

We model the cpu load for a configuration using a neural network.

- Input: activated functions (e.g., $f_1 = 1, f_2 = 0, \ldots$)
- Output: estimated cpu load.



- Important: we do not assume monotonicity (e.g., activating a function *can reduce* load)

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Problem & Opportunities

We obatin such a NN, which has many inputs and is a black box model.

- Hard to check all combinations.
- Not clear the cause of overload.

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Identifying insignficant inputs.

- Are some inputs *insignificant*?
- Can we remove them?
- What is the effect on the network? Can it be bounded?

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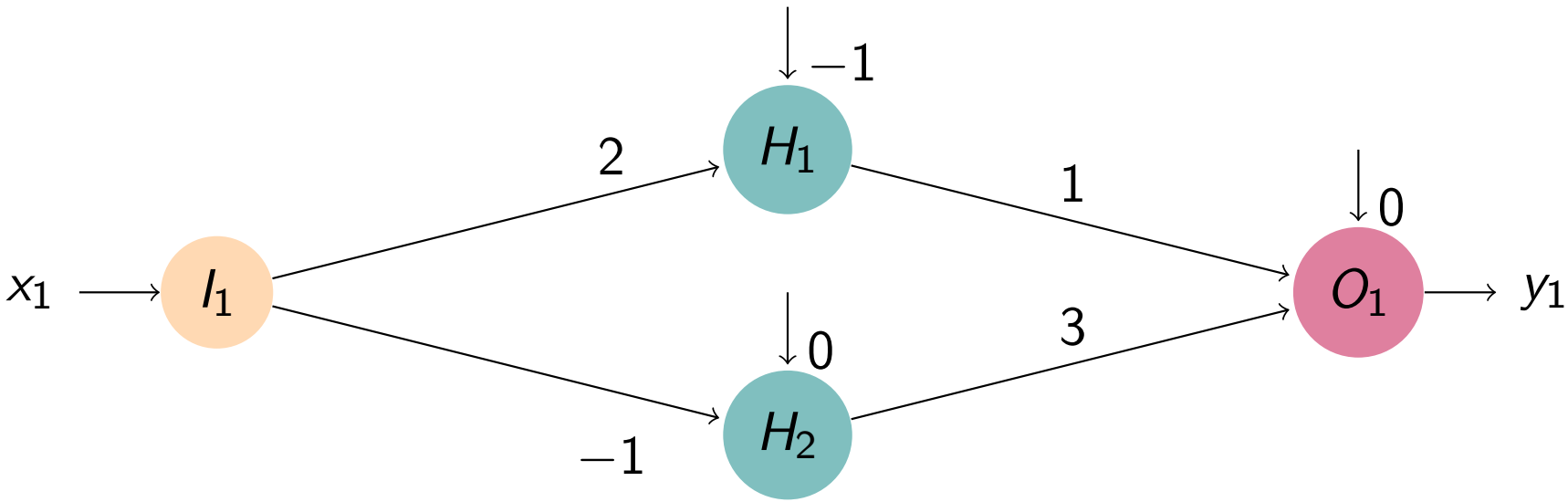
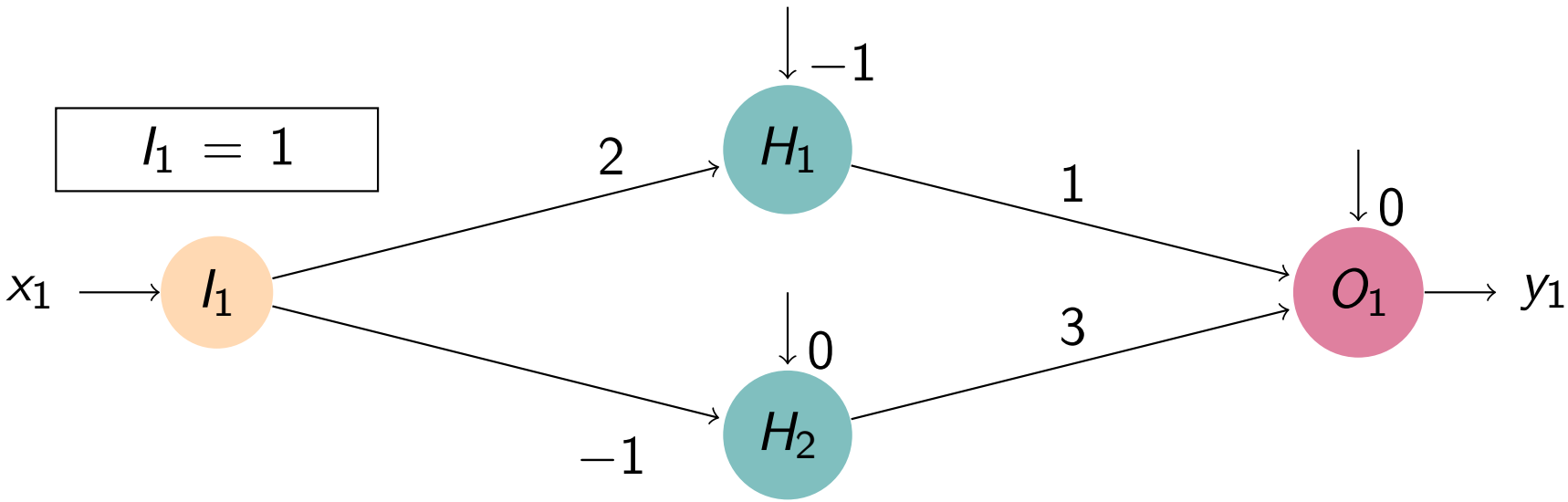
Neural Networks

Neural Networks



Neural Networks



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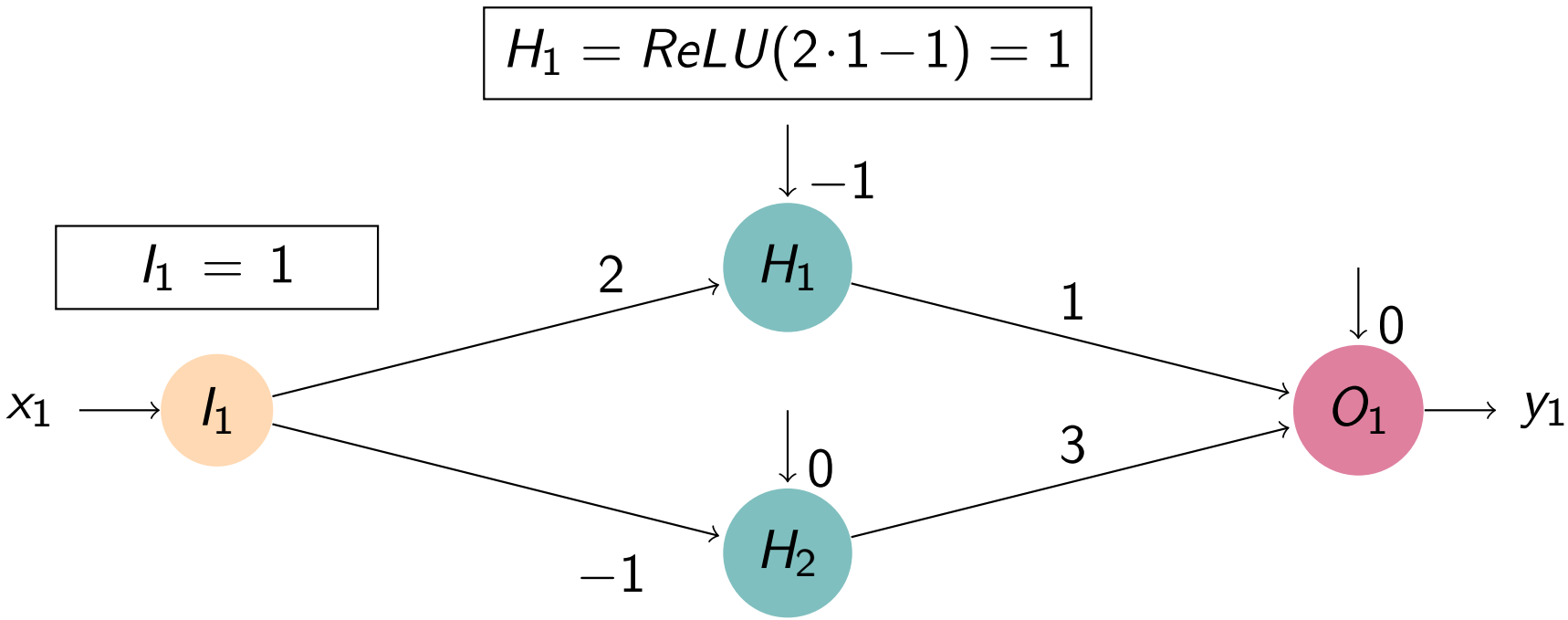

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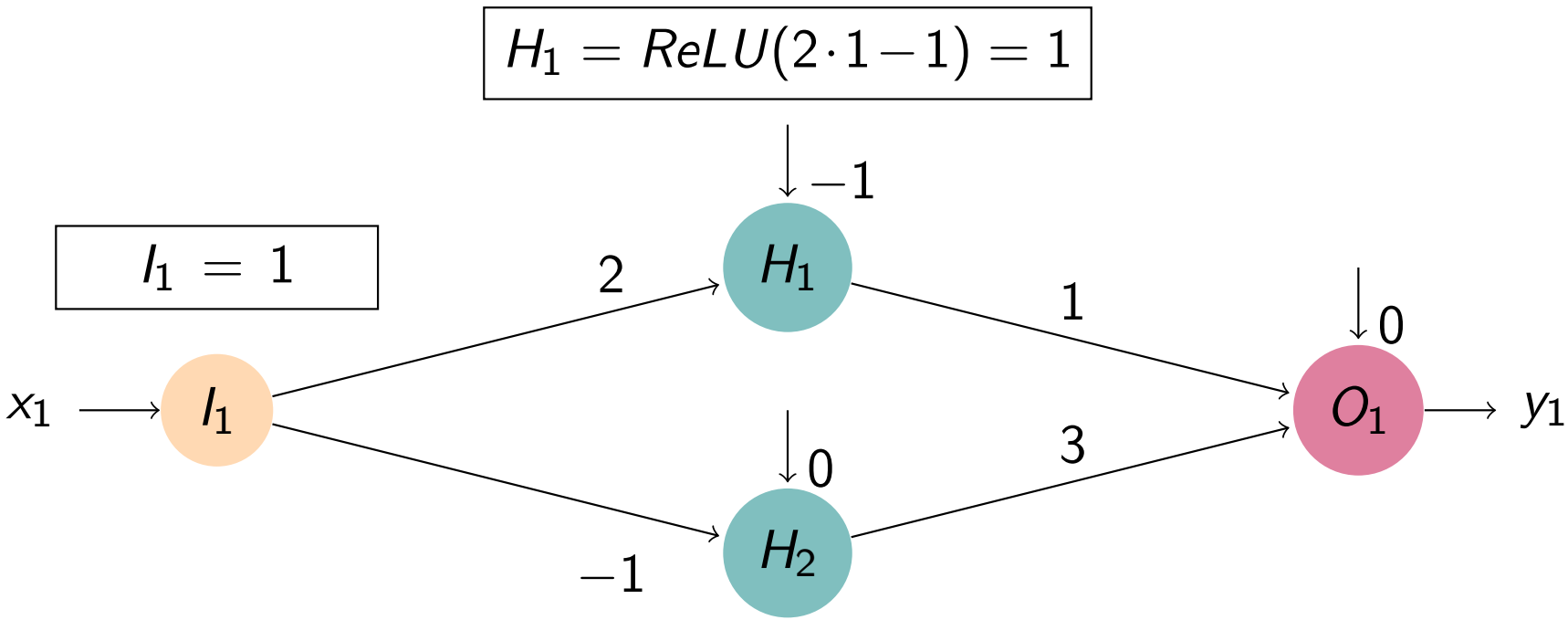
Neural Networks



$$H_2 = ReLU(-1 \cdot 1 + 0) = 0$$

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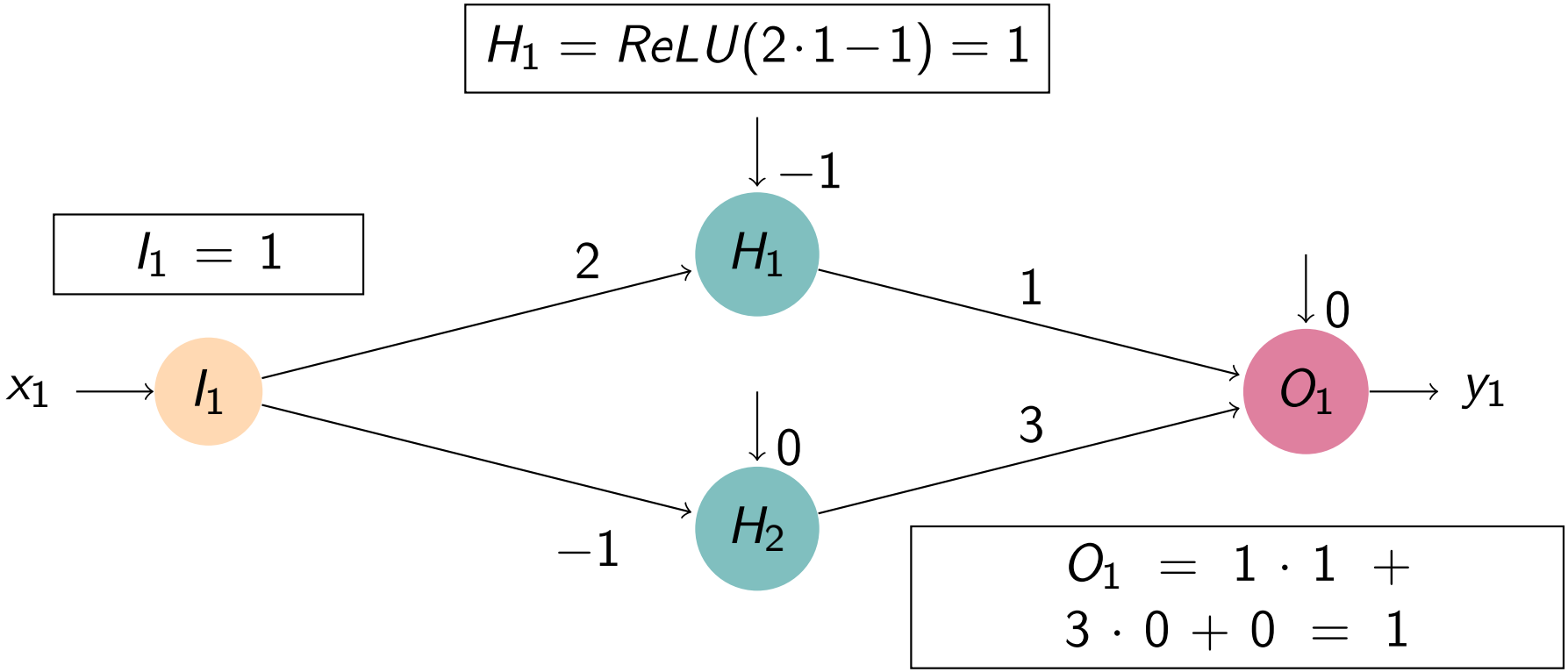
Neural Networks



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NN Verification

Tools for verifying NN.

- Marabou can check properties of NN
- For example: establish (linear) upper bound on output node
- Under some constraints on input

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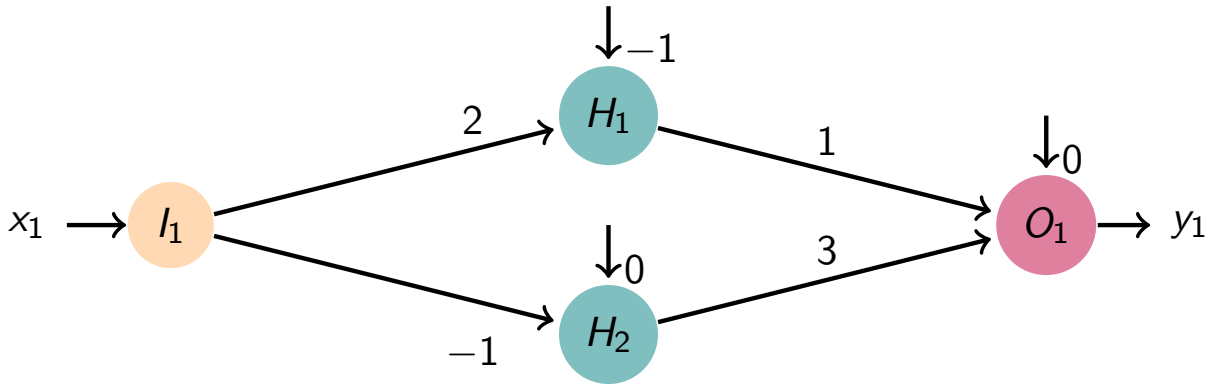

For example, Marabou can show that $O_1(x) \leq 1$, for all $x \in [0, 1]$.

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Abstracting for Over-approximation

Abstracting for Over-approximation

- We are considering the establishment of an *upper bound*.
- This means, we can replace a network $\mathcal{NN}$ with an abstracting $\mathcal{NN}'$ if:

$$\forall x. \mathcal{NN}(x) \leq \mathcal{NN}'(x)$$

- If we find a upper bound U for $\mathcal{NN}'(x)$, then:

$$\forall x. \mathcal{NN}(x) \leq \mathcal{NN}'(x) \leq U$$

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Abstraction idea presented in:
Yizhak Yisrael Elboher, Justin Gottschlich, and Guy Katz. "An abstraction-based framework for neural network verification". In: *Computer Aided Verification: 32nd International Conference, CAV 2020, Los Angeles, CA, USA, July 21–24, 2020, Proceedings, Part I 32*. Springer. 2020, pp. 43–65:

Classify nodes into increasing and decreasing nodes

We demonstrate through an example.

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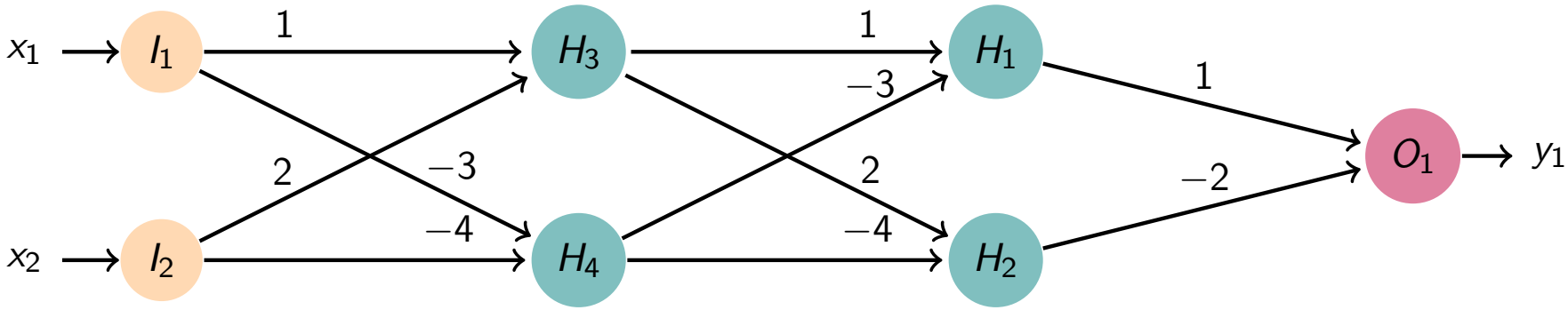
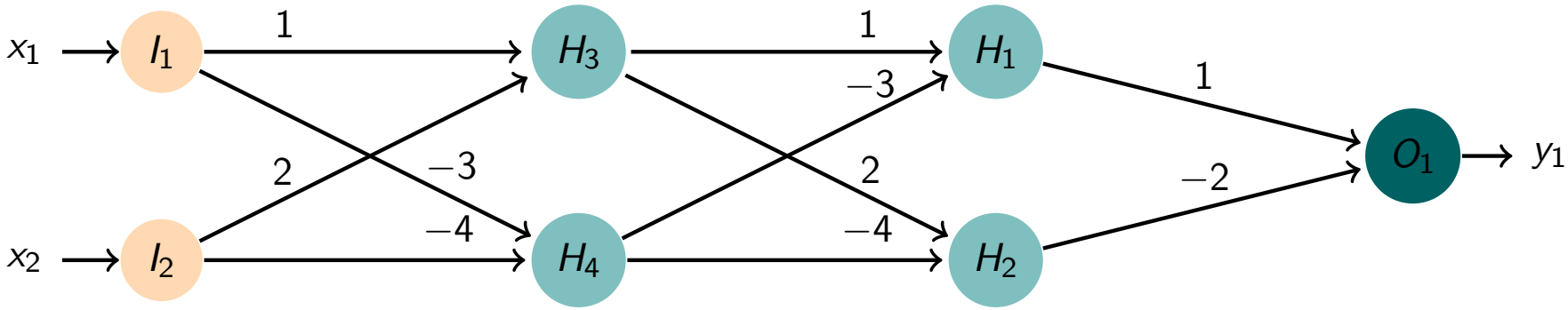
Abstraction Example



Abstraction Example



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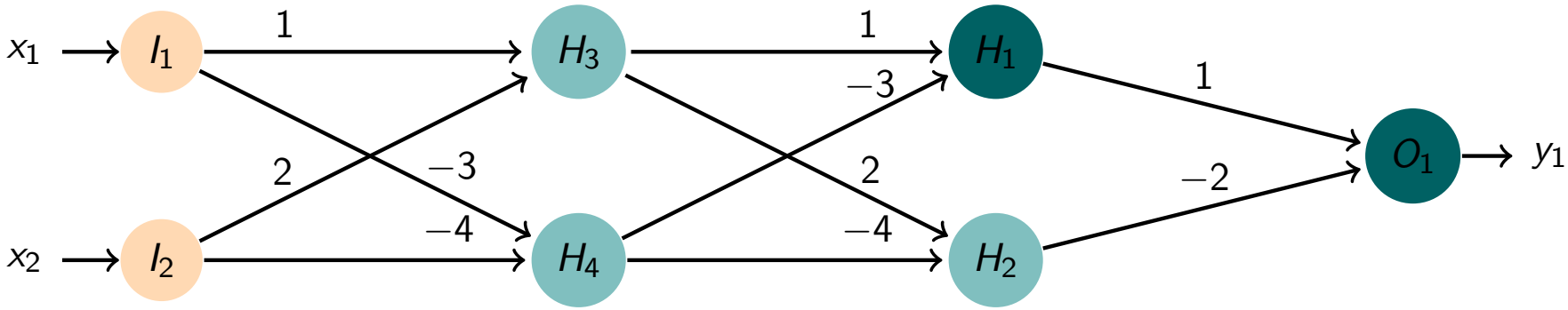

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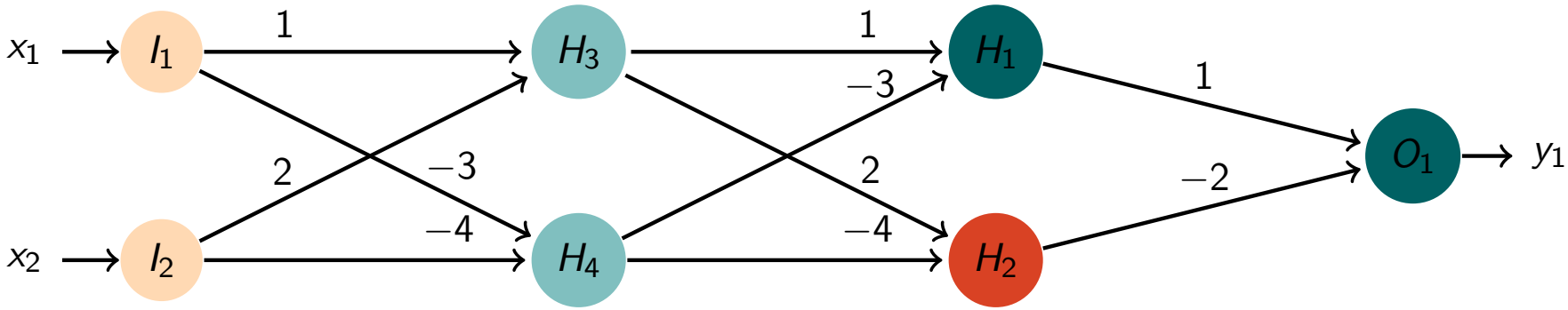

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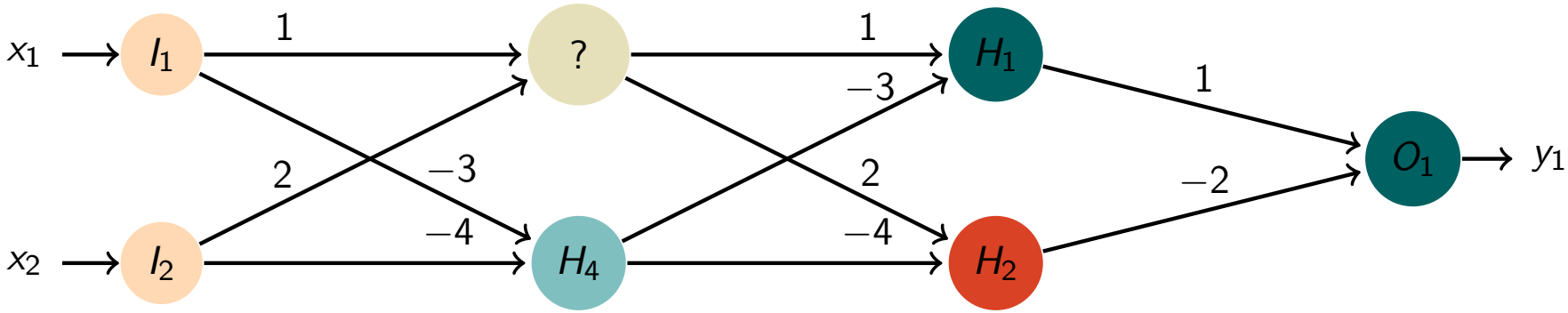
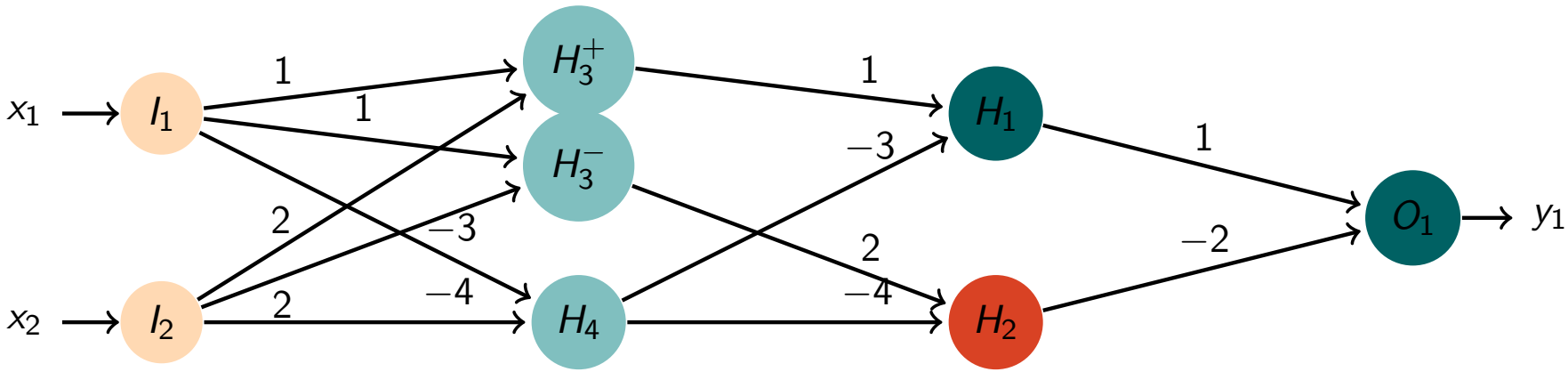


Let's copy the node!

Abstraction Example



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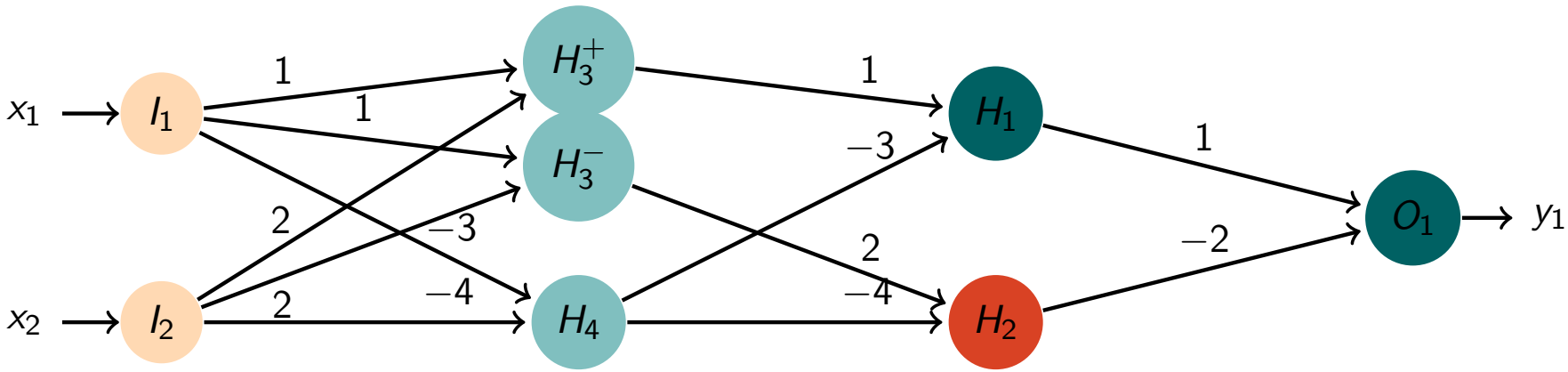

This computes the exact same function.

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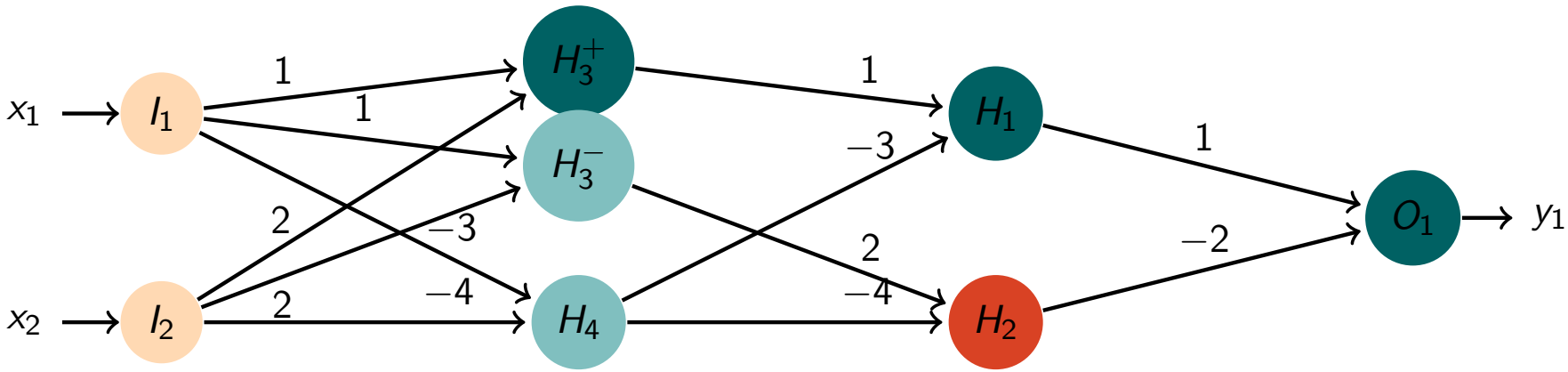

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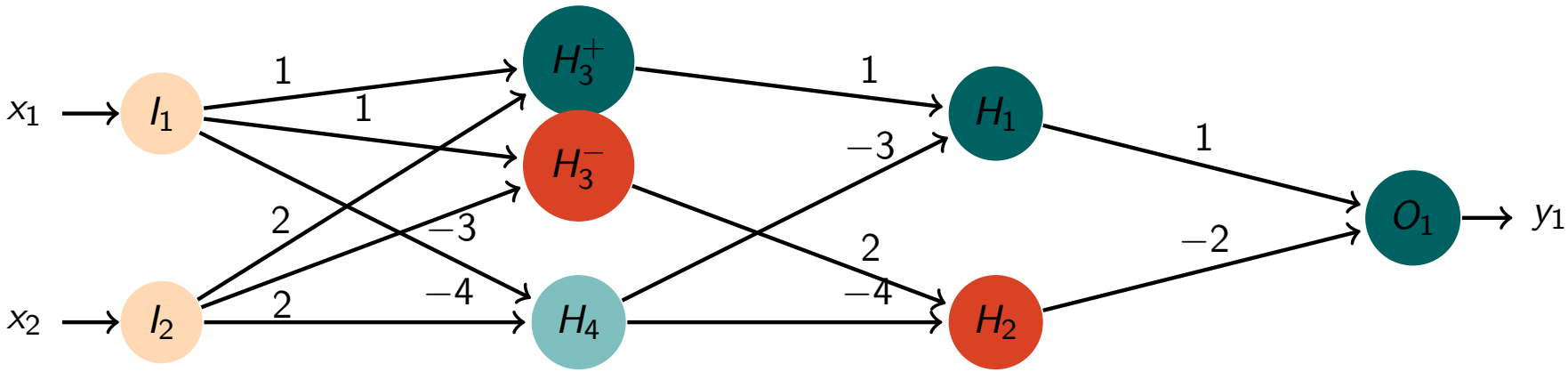

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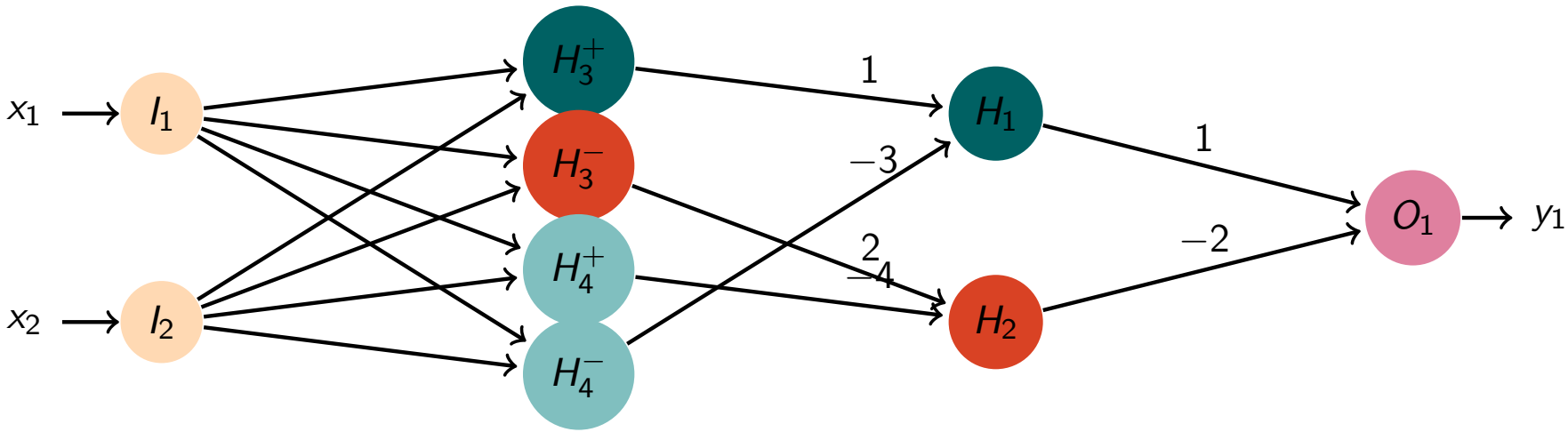

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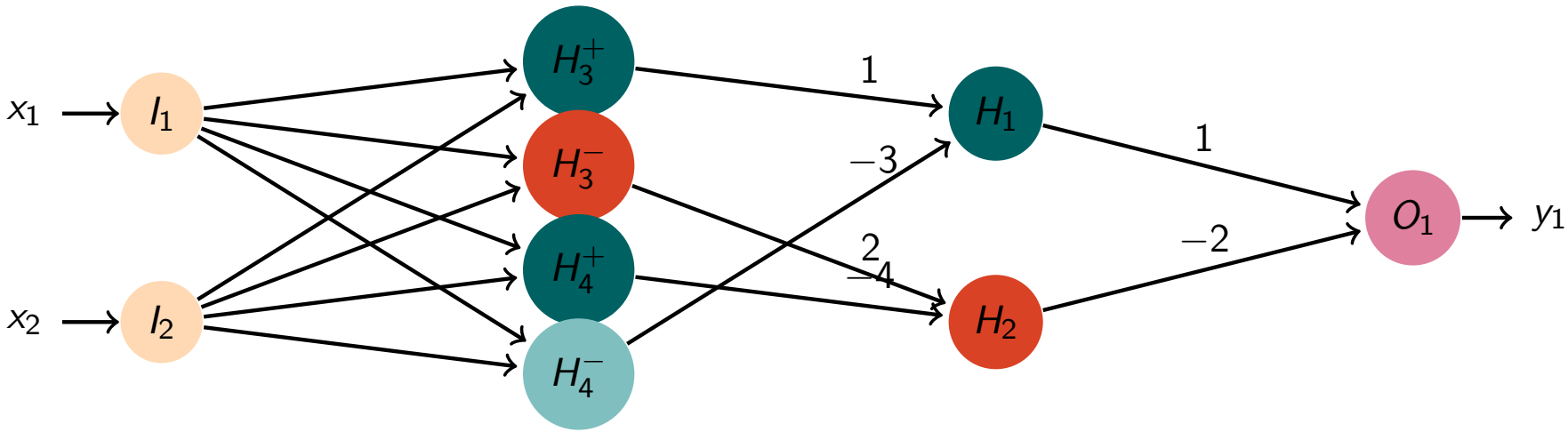

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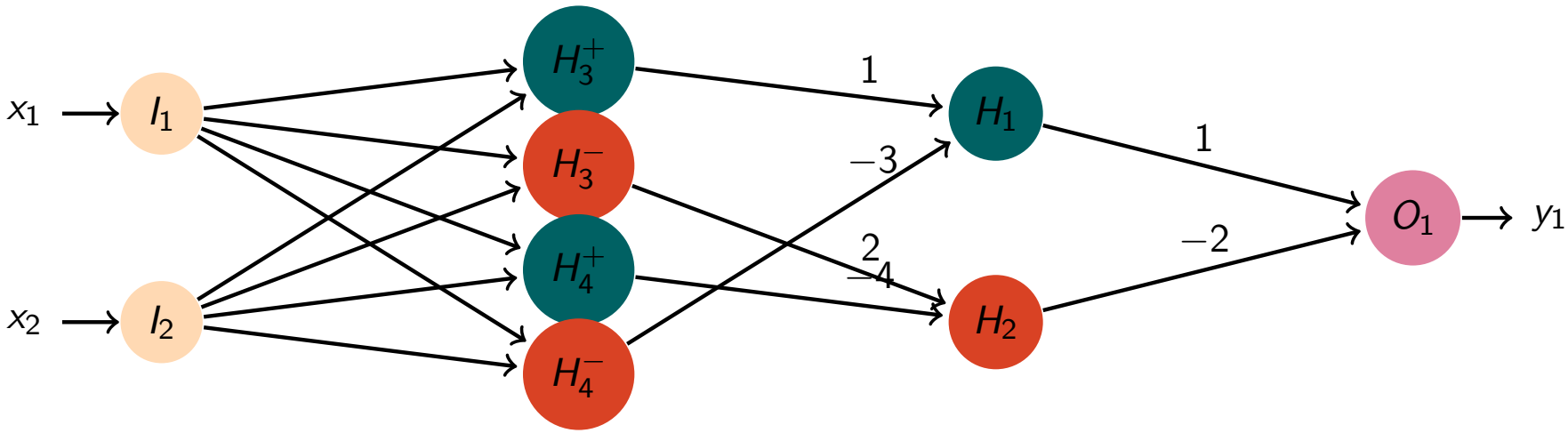

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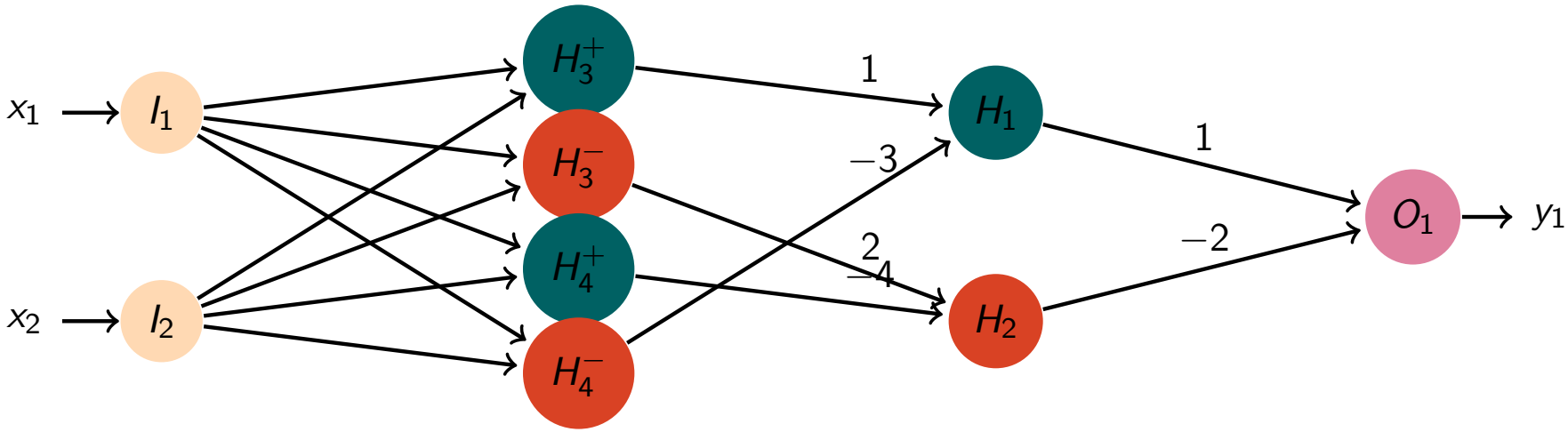

This computes the exact same function.

- In this manner we can color each hidden node in network
- Might double number of nodes
- Our proposal: extend this to input layer

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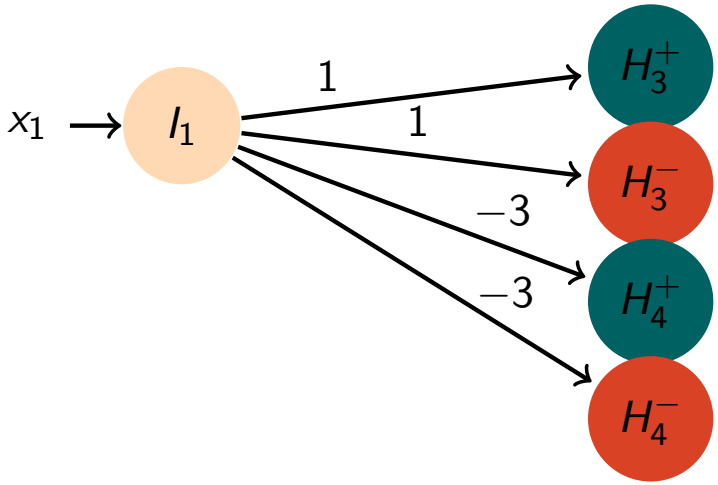
Abstract input layer



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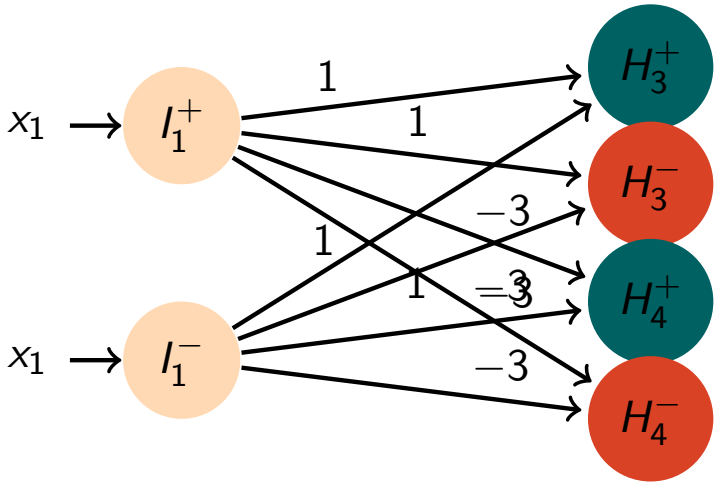

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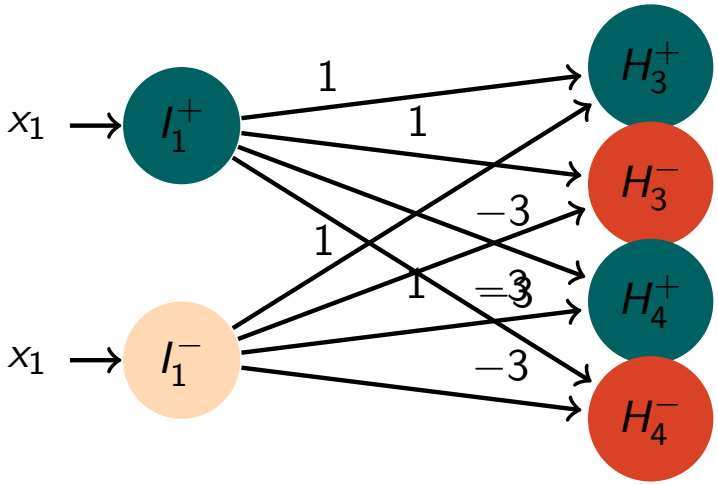

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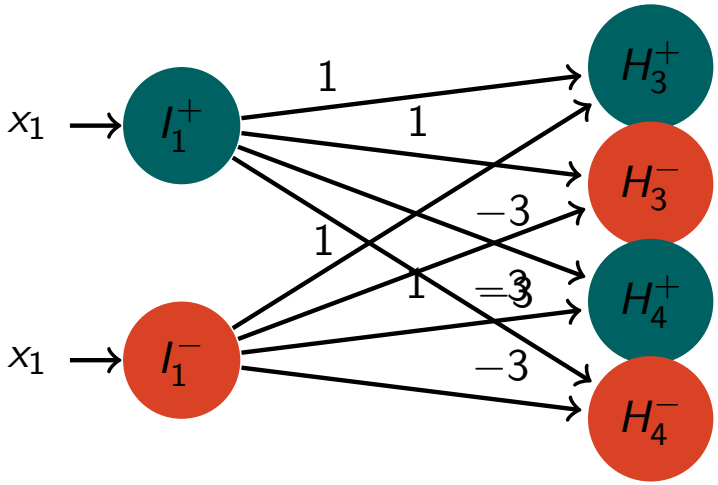

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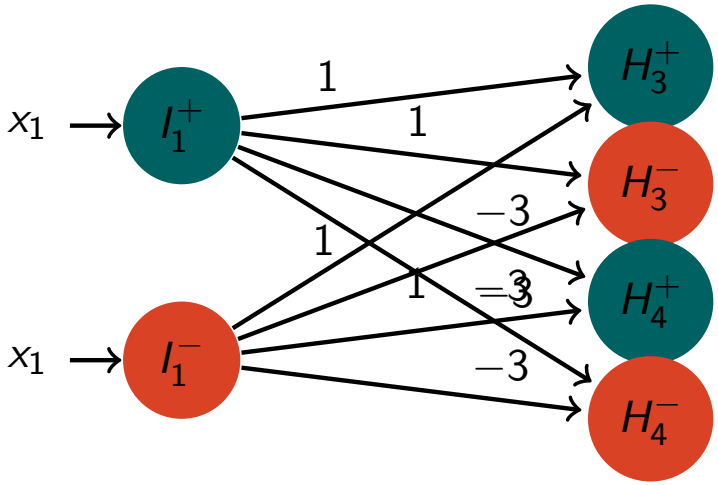

- I_1^+ is the increasing component of I_1
- We set it to the maxmimum value of x_1
- Similarily, I_1^- is set to the minimum value of x_1

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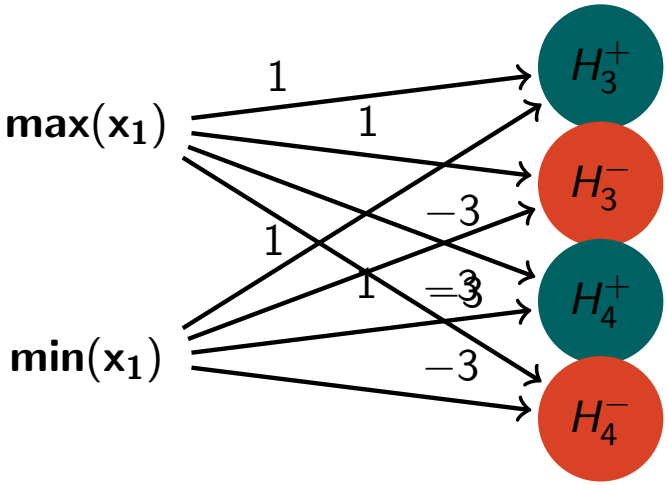

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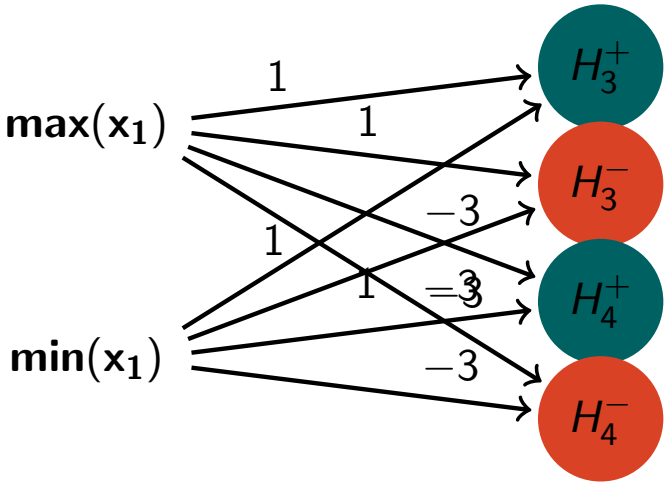

- I_1^+ is the increasing component of I_1
- We set it to the maxmimum value of x_1
- Similarily, I_1^- is set to the minimum value of x_1
- Now we have removed input I_1!

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Removing one input

- In this manner we can remove an input and I_i and create an over-approximating network $\mathcal{NN}_i^+$, s.t.

$$\mathcal{NN}(x) \leq \mathcal{NN}_i^+(x')$$

  for all x, x' (where x' is x with x_i removed)
- $\mathcal{NN}_i^+$ is throwing x_i out of the window, but *at what cost*?

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Bounding approximation cost

- In the same manner as we create $\mathcal{NN}_i^+$, we can create an *under-approximating* network $\mathcal{NN}_i^-$.
- We can use the combination to bound the approximation cost.

Bounding approximation cost

$$\mathcal{NN}_i^+$$

Bounding approximation cost

$\mathcal{NN}_i^+$

$\mathcal{NN}_i^-$

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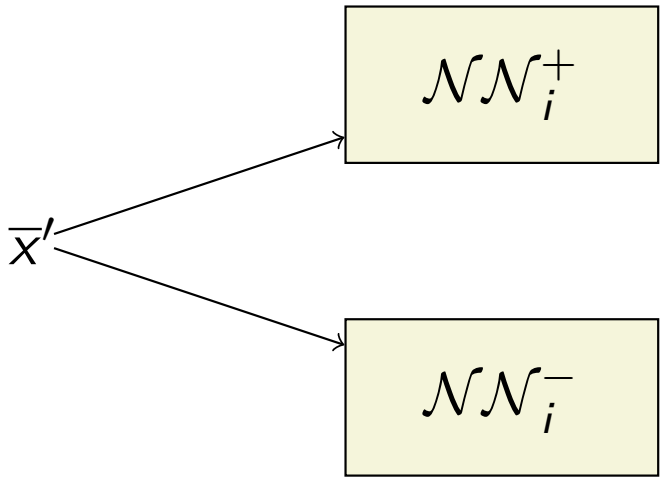
Bounding approximation cost



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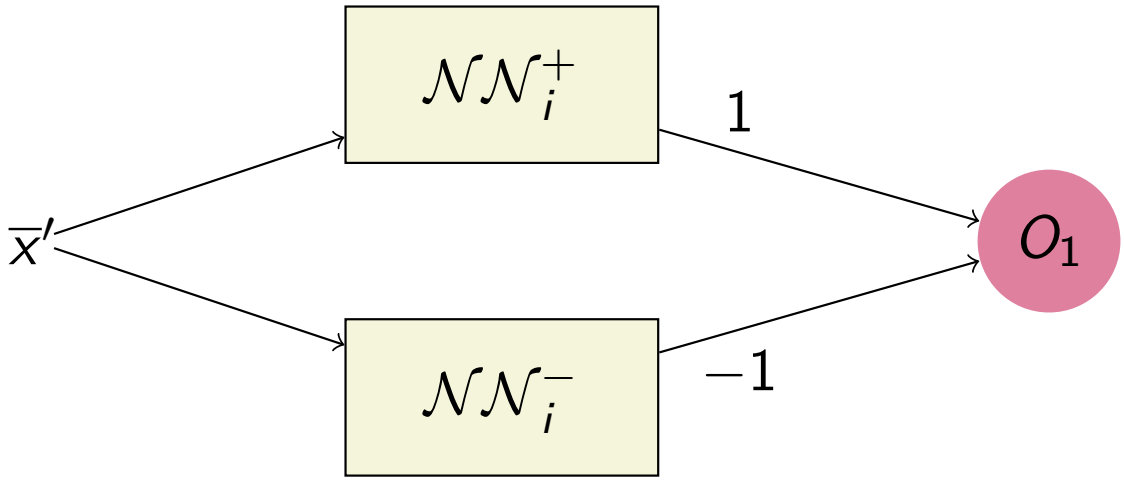

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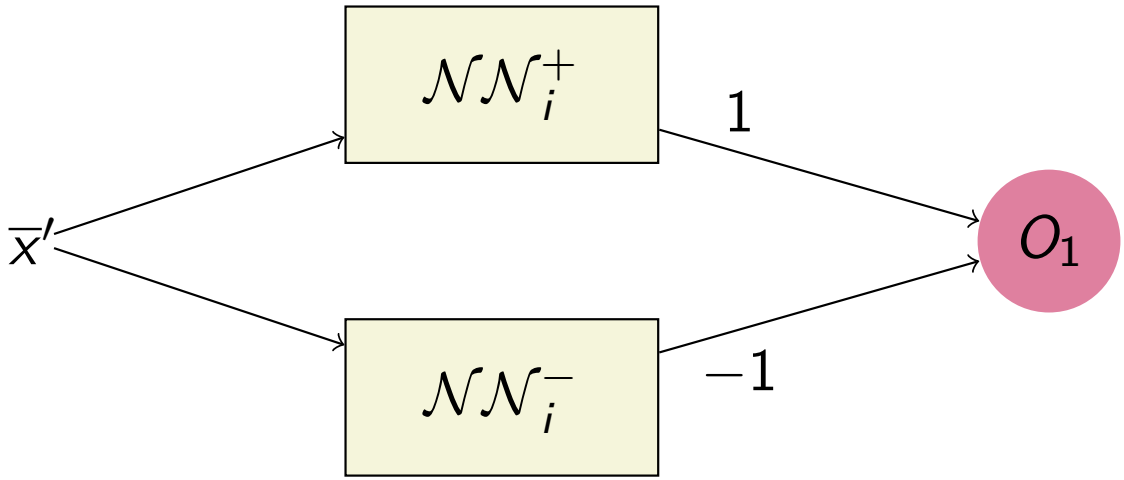

- O_1 gives the an over-approximation of the *maximum* effect of x_i.
- Let's call this network $\mathcal{NN}_i^?$.

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Bounding approximation cost

- We have a network $\mathcal{NN}_i^?$ giving the over-approximation.
- We use Marabou to establish an upper bound:

$$\mathcal{NN}_i^? \leq L$$

  for some limit L.
- If true, x_i affects output of network by at most L
- If L small, we call x_i *insignficant*.

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- For a particular subset of F (a configuration), we can measure a CPU load.
- Question: for what configurations is there risk for an overload?

- We can remove an input f_i and (maybe) find a limit L.
- This means that function f_i can be ignored if we add L to the final load.

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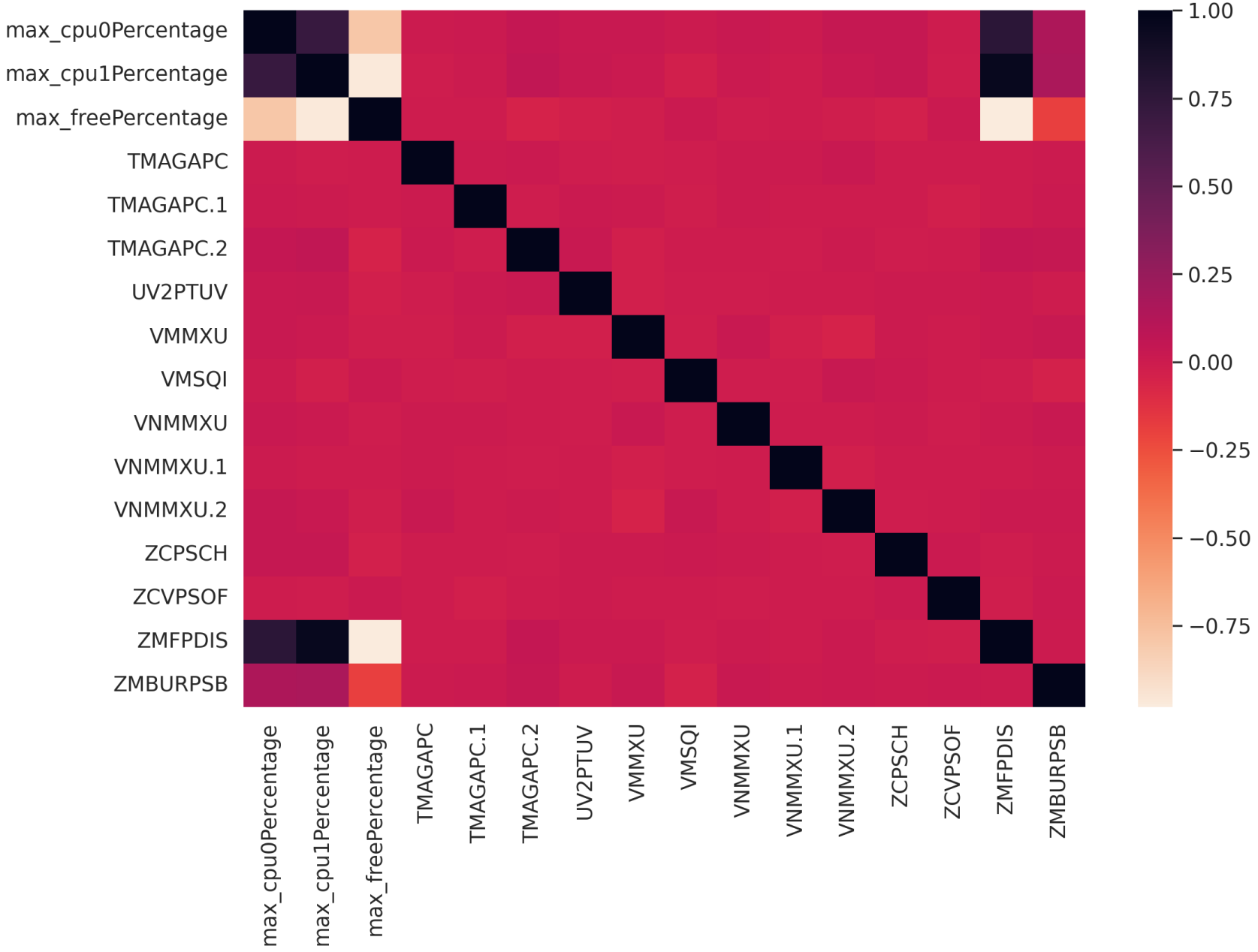
(Simple) Validation

- We harvest data from embedded system
- Roughly 3000 samples for 13 functions
- Train neural network (10 hidden nodes)

Correlation



Our results

- We try to idenity each function and see if it is *insignificant*
- A limit of 2 identifies $\mathrm{ZMFPDIS}$ and $\mathrm{ZMBURPSB}$ as significant inputs.
- A limit of 1 only identifies four insignficant inputs.
- A limit of 0.5 identifies no insignficant inputs.

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Conclusions

- Process non-deterministic
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Conclusions

- Process non-deterministic
- Finding bound requires a guess

- We can extract information of model (but is model correct?)
- We can reduce complexity (inputs) of the model, by adding a constant to the output

- Continue validation
- Consider tuning training of network

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References

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